Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5814812485

ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **May/June 2020**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

$x = 0$

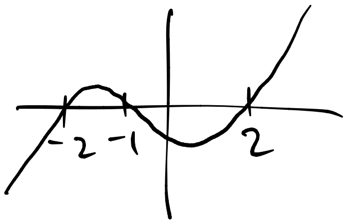
1 On the axes below, sketch the graph of $y = |(x-2)(x+1)(x+2)|$ showing the coordinates of the points where the curve meets the axes. [3]

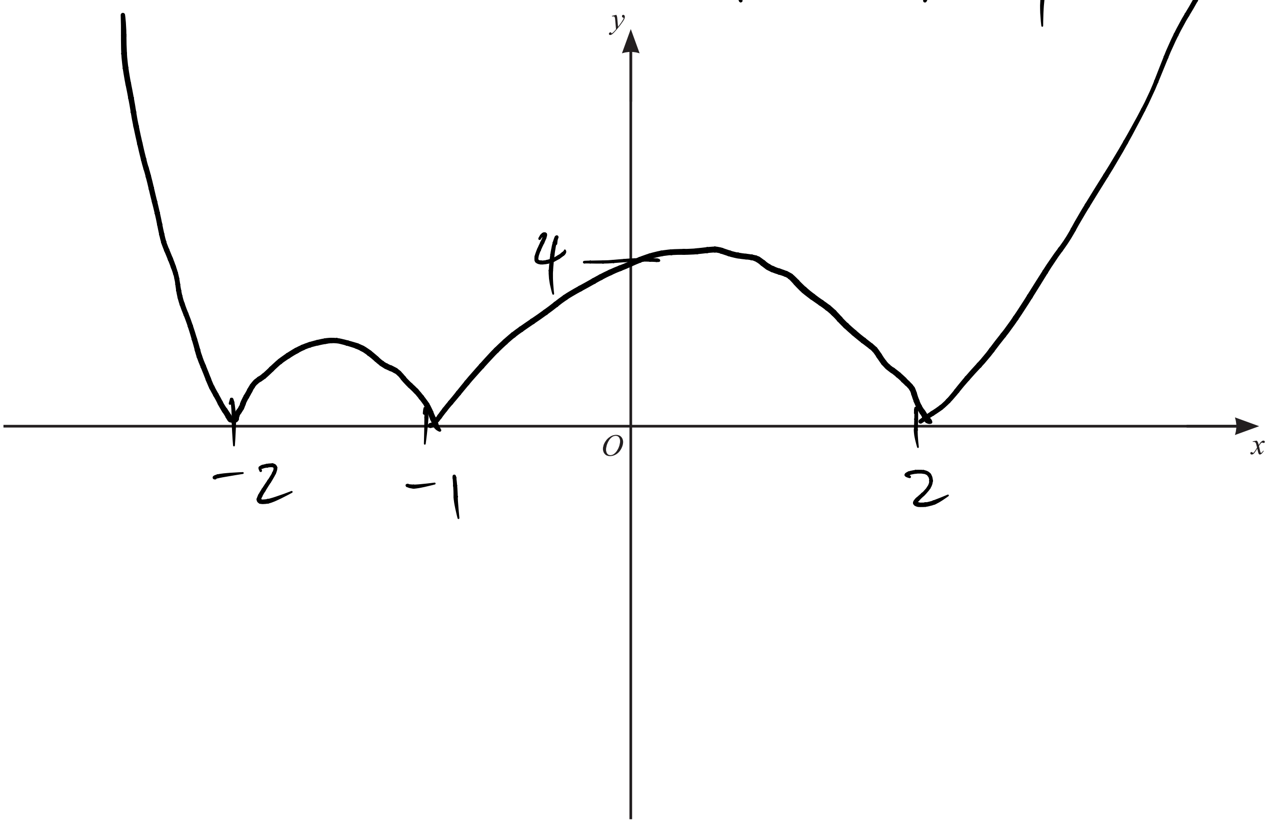
2 −1 −2

$y = |(-2)(1)(2)|$

$y = |-4|$

$y = 4$





2 The volume, V, of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

The radius, r cm, of a sphere is increasing at the rate of $0.5\,\text{cm}\,\text{s}^{-1}$. Find, in terms of π, the rate of change of the volume of the sphere when $r = 0.25$. [4]

$$\frac{dV}{dt} = ? \qquad \frac{dr}{dt} = 0.5 \text{ cm/s}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \qquad \frac{dV}{dr} = 4\pi r^2$$

$$= 4\pi r^2 \times 0.5$$

$$= 2\pi r^2$$

$$r = 0.25$$

$$= 2\pi(0.25)^2$$

$$= \boxed{0.125\pi \text{ cm}^3/\text{s}}$$

3 (a) Find the first 3 terms in the expansion of $\left(4-\frac{x}{16}\right)^6$ in ascending powers of x. Give each term in its simplest form. [3]

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \ldots$$

$$\left(4-\frac{x}{16}\right)^6 = 4^6 + \binom{6}{1}4^5\left(-\frac{x}{16}\right) + \binom{6}{2}4^4\left(-\frac{x}{16}\right)^2$$

$$= \boxed{4096 - 384x + 15x^2}$$

(b) Hence find the term independent of x in the expansion of $\left(4-\frac{x}{16}\right)^6\left(x-\frac{1}{x}\right)^2$. [3]

$$(4096 - 384x + 15x^2)\left(x^2 - 2 + \frac{1}{x^2}\right)$$

$$= -2(4096) + 15$$

$$= \boxed{-8177}$$

4 **(a)** **(i)** Find how many different 5-digit numbers can be formed using the digits 1, 2, 3, 5, 7 and 8, if each digit may be used only once in any number. [1]

_ _ _ _ _ $\quad {}^6P_5 = \boxed{720}$

(ii) How many of the numbers found in **part (i)** are not divisible by 5? [1]

_ _ _ _ 5 $\quad {}^5P_4 = 120 \quad 720 - 120 = \boxed{600}$

(iii) How many of the numbers found in **part (i)** are even and greater than 30 000? [4]

1 $\quad$ 3 _ _ _ 2 $\quad$ 7 _ _ _ 2 $\quad {}^4P_3 = 24$

2 $\quad$ 3 _ _ _ 8 $\quad$ 7 _ _ _ 8 $\quad 24 \times 7 = \boxed{168}$

3 $\quad$ 5 _ _ _ 2 $\quad$ 8 _ _ _ 2

4 $\quad$ 5 _ _ _ 8

(b) The number of combinations of n items taken 3 at a time is 6 times the number of combinations of n items taken 2 at a time. Find the value of the constant n. [4]

$$\binom{n}{3} = 6\binom{n}{2} \qquad \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$n! = n \times (n-1)!$$

$$\frac{\cancel{n!}}{3!(n-3)!} = 6\frac{\cancel{n!}}{2!(n-2)!}$$

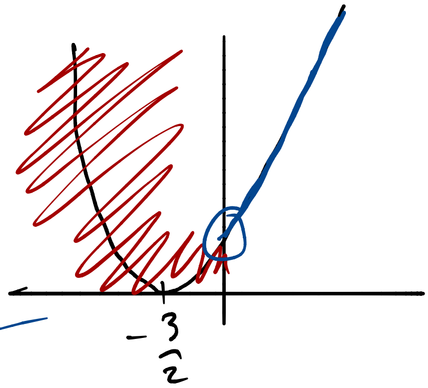
$$\frac{1}{6\cancel{(n-3)!}} = \frac{6}{2(n-2)\cancel{(n-3)!}}$$

$$\frac{1}{6} = \frac{3}{n-2}$$

$$n-2 = 18$$

$$\boxed{n = 20}$$

5 $\quad \mathrm{f}: x \mapsto (2x+3)^2 \quad \text{for } x > 0$



(a) Find the range of f. [1]

$f(0) = (2(0)+3)^2$

$= 9$

$\boxed{f(x) > 9}$

(b) Explain why f has an inverse. [1]

$\because$ f is one to one under the domain

(c) Find f^{-1}. [3]

$x = (2f(x)+3)^2$

$2f(x)+3 = \sqrt{x}$

$2f(x) = \sqrt{x} - 3$

$\boxed{f(x) = \dfrac{\sqrt{x}-3}{2}}$

(d) State the domain of f^{-1}. [1]

$\boxed{x > 9}$

f $\quad$ f^{-1}

dom $\quad$ dom

rang $\quad$ rang

(e) Given that $\mathrm{g}: x \mapsto \ln(x+4)$ for $x > 0$, find the exact solution of $\mathrm{fg}(x) = 49$. [3]

$f(g(x)) = (2g(x)+3)^2 = 49$ $\qquad e^{\ln(x)} = x$

$2g(x)+3 = 7$

$g(x) = 2$

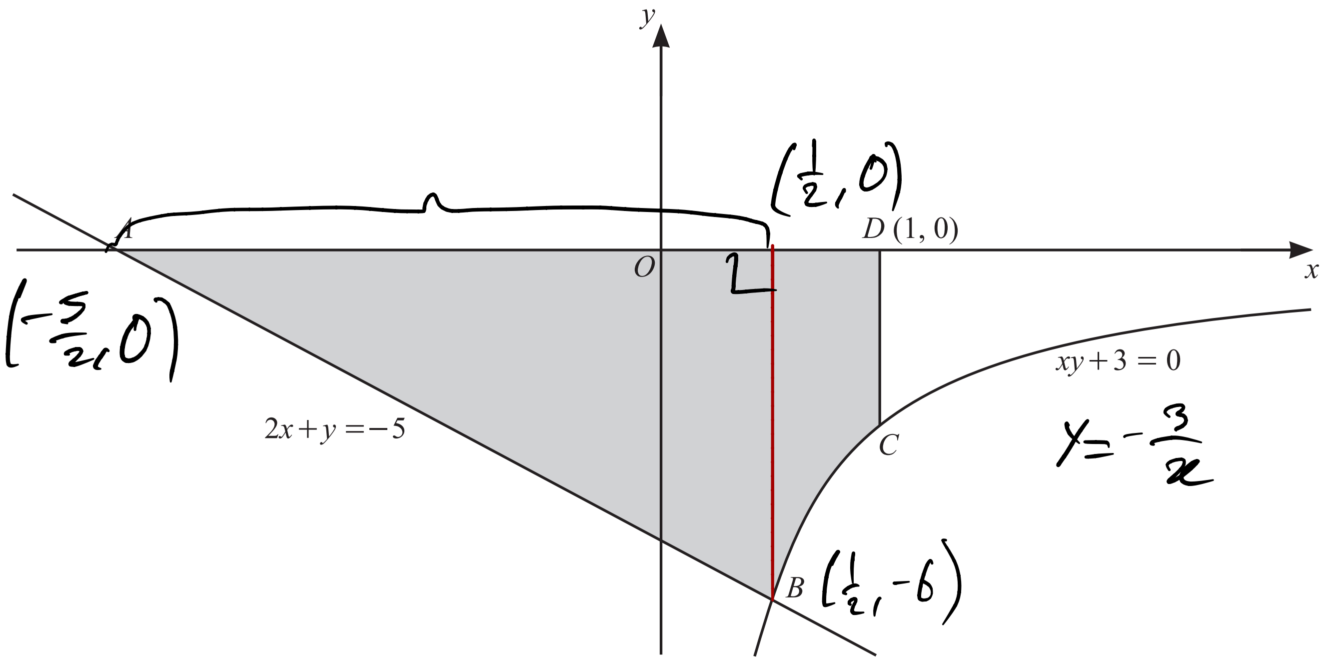
$\ln(x+4) = 2$

$\cancel{e}^{\cancel{\ln}(x+4)} = e^2$

$x+4 = e^2$

$\boxed{x = e^2 - 4}$

6



The diagram shows the straight line $2x+y=-5$ and part of the curve $xy+3=0$. The straight line intersects the x-axis at the point A and intersects the curve at the point B. The point C lies on the curve. The point D has coordinates $(1, 0)$. The line CD is parallel to the y-axis.

(a) Find the coordinates of each of the points A and B. [3]

$2x = -5$

$x = -\frac{5}{2}$

$\boxed{A\left(-\frac{5}{2}, 0\right) \quad B\left(\frac{1}{2}, -6\right)}$

$2x + y = -5$

$xy + 3 = 0$

$\to x = \frac{-3}{y}$

$2\left(-\frac{3}{y}\right) + y = -5$

$x = \frac{-3}{-6} = \frac{1}{2}$

$-\frac{6}{y} + y = -5$

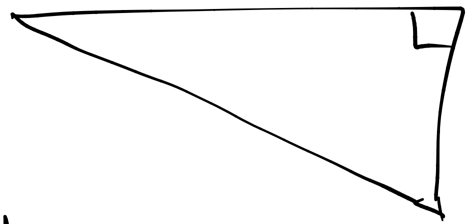
$-6 + y^2 + 5y = 0$ $\quad y = -6$

$y^2 + 5y - 6 = 0$ $\quad \cancel{y = 1}$

$(y+6)(y-1) = 0$

(b) Find the area of the shaded region, giving your answer in the form $p + \ln q$, where p and q are positive integers. [6]

$\frac{1}{2} - \left(-\frac{5}{2}\right) = 3$



$\frac{1}{2}(3)(6) = 9$

$$\int_{\frac{1}{2}}^{1} -\frac{3}{x}\,dx$$

$$= -3\int_{\frac{1}{2}}^{1} \frac{1}{x}\,dx$$

$$= -3\left[\ln x\right]_{\frac{1}{2}}^{1}$$

$$= -3\left(\overset{0}{\cancel{\ln(1)}} - \ln\left(\tfrac{1}{2}\right)\right)$$

$$= 3\ln(2^{-1})$$

$$= \cancel{-}3\ln(2)$$

$$3\ln(2)$$

$9 + 3\ln(2)$

$\boxed{9 + \ln(8)}$

7 **(a)** Given that $y = \overset{u}{(x^2 - 1)}\overset{v}{\sqrt{5x+2}}$, show that $\dfrac{dy}{dx} = \dfrac{Ax^2 + Bx + C}{2\sqrt{5x+2}}$, where A, B and C are integers. [5]

$(uv)' = u'v + uv'$

$\dfrac{d}{dx}\sqrt{5x+2} = \dfrac{5}{2\sqrt{5x+2}}$

$$\frac{dy}{dx} = 2x\sqrt{5x+2} + (x^2-1)\frac{5}{2\sqrt{5x+2}}$$

$$= \frac{4x(5x+2) + (x^2-1)(5)}{2\sqrt{5x+2}}$$

$$= \frac{20x^2 + 8x + 5x^2 - 5}{2\sqrt{5x+2}}$$

$$= \boxed{\frac{25x^2 + 8x - 5}{2\sqrt{5x+2}}}$$

(b) Find the coordinates of the stationary point of the curve $y = (x^2 - 1)\sqrt{5x+2}$, for $x > 0$. Give each coordinate correct to 2 significant figures. [3]

0.31 0.31

$$\frac{dy}{dx} = 0$$

$$\frac{25x^2 + 8x - 5}{\cancel{2\sqrt{5x+2}}} = 0$$

$$25x^2 + 8x - 5 = 0$$

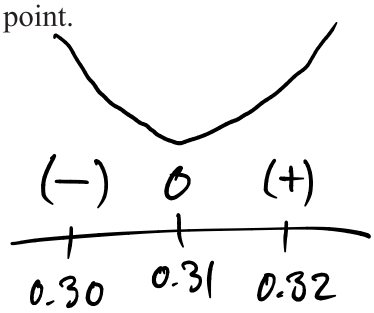
$x_1 = 0.31$ $\cancel{x_2} = -0.63$

$y = -1.7$

$\boxed{(0.31, -1.7)}$

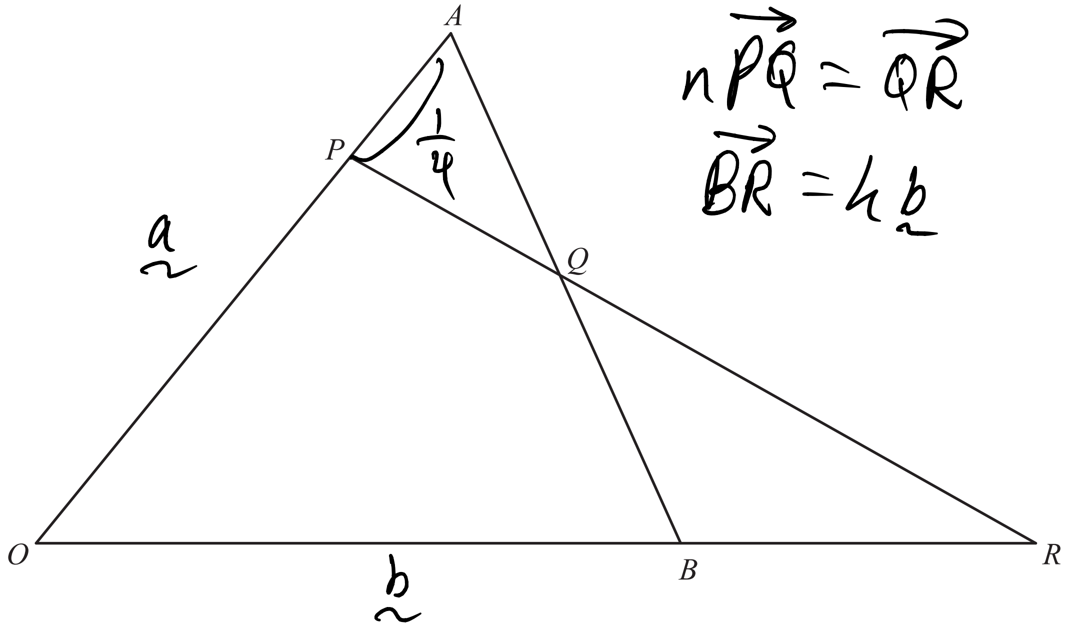
(c) Determine the nature of this stationary point. [2]

$$\frac{dy}{dx} = \frac{25x^2 + 8x - 5}{2\sqrt{5x+2}}$$



$\boxed{\text{Minimum}}$

8



The diagram shows a triangle OAB such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point P lies on OA such that $OP = \frac{3}{4}OA$. The point Q is the mid-point of AB. The lines OB and PQ are extended to meet at the point R. Find, in terms of **a** and **b**,

(a) $\overrightarrow{AB}$, [1]

$\boxed{\underset{\sim}{b} - \underset{\sim}{a}}$

(b) $\overrightarrow{PQ}$. Give your answer in its simplest form. [3]

$\frac{1}{4}\underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$

$\frac{1}{4}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

$\boxed{\frac{1}{2}\underset{\sim}{b} - \frac{1}{4}\underset{\sim}{a}}$

It is given that $n\overrightarrow{PQ} = \overrightarrow{QR}$ and $\overrightarrow{BR} = k\mathbf{b}$, where n and k are positive constants.

(c) Find $\overrightarrow{QR}$ in terms of n, **a** and **b**. [1]

$$\boxed{\overrightarrow{QR} = n\left(\frac{1}{2}\underset{\sim}{b} - \frac{1}{4}\underset{\sim}{a}\right)}$$

(d) Find $\overrightarrow{QR}$ in terms of k, **a** and **b**. [2]

$$\boxed{\frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a}) + k\underset{\sim}{b}}$$

(e) Hence find the value of n and of k. [3]

$$n\left(\frac{1}{2}\underset{\sim}{b} - \frac{1}{4}\underset{\sim}{a}\right) = \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a}) + k\underset{\sim}{b}$$

$$\frac{n}{2}\underset{\sim}{b} - \frac{n}{4}\underset{\sim}{a} = \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a} + k\underset{\sim}{b}$$

$$\frac{n}{2} = \frac{1}{2} + k$$

$$1 = \frac{1}{2} + k$$

$$-\frac{n}{4} = -\frac{1}{2}$$

$$\boxed{k = \frac{1}{2} \quad n = 2}$$

$$n = 2$$

9 **(a)** A particle P moves in a straight line such that its displacement, x m, from a fixed point O at time t s is given by $x = 10\sin 2t - 5$.

(i) Find the speed of P when $t = \pi$. [1]

$$\frac{dx}{dt} = 20\cos(2t)$$

$$t = \pi$$

$$V = \frac{dx}{dt} = \boxed{20}\ m/s$$

(ii) Find the value of t for which P is first at rest. [2]

$$\cancel{20}\cos(2t) = 0$$

$$2t = \cos^{-1}(0)$$

$$2t = \frac{\pi}{2} \longrightarrow \boxed{t = \frac{\pi}{4}\ s}$$

(iii) Find the acceleration of P when it is first at rest. [2]

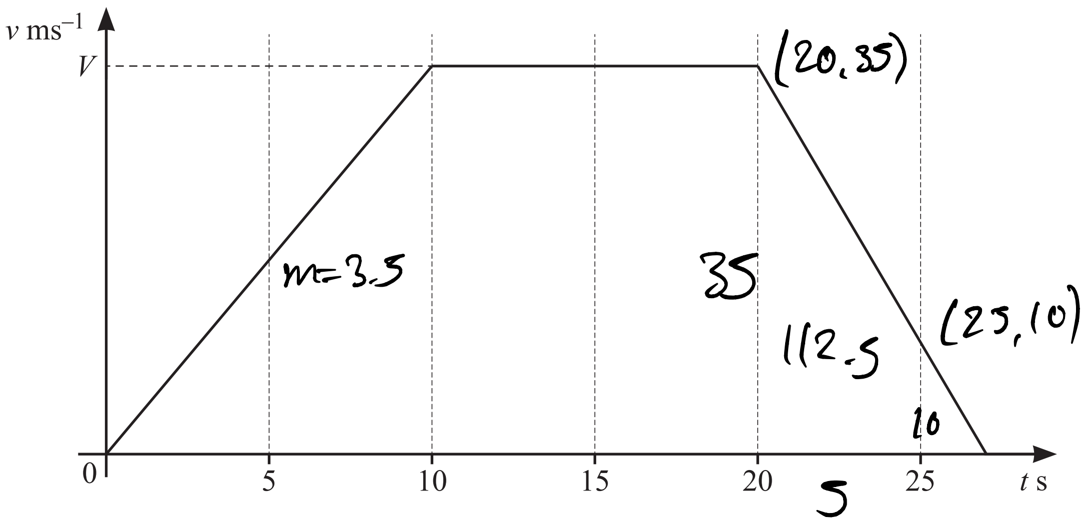
$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = -40\sin(2t)$$

$$t = \frac{\pi}{4}$$

$$-40\sin\left(\frac{\pi}{2}\right) = \boxed{-40}\ m/s^2$$

$$1$$

(b)



The diagram shows the velocity–time graph for a particle Q travelling in a straight line with velocity $v\,\mathrm{ms}^{-1}$ at time t s. The particle accelerates at $3.5\,\mathrm{ms}^{-2}$ for the first 10 s of its motion and then travels at constant velocity, $V\,\mathrm{ms}^{-1}$, for 10 s. The particle then decelerates at a constant rate and comes to rest. The distance travelled during the interval $20 \leqslant t \leqslant 25$ is 112.5 m.

(i) Find the value of V. [1]

$V = 3.5t$

$t = 10$

$\boxed{V = 3.5 \times 10 = 35\,\mathrm{m/s}}$

(ii) Find the velocity of Q when $t = 25$. [3]

$\dfrac{5(35+x)}{2} = 112.5$

$5(35+x) = 225$

$35 + x = 45$

$\boxed{x = 10\,\mathrm{m/s}}$

(iii) Find the value of t when Q comes to rest. [3]

$m = \dfrac{35-10}{20-25} = \dfrac{25}{-5} = -5$

$y = m(x - x_1) + y_1$

$y = -5(x-25)+10$

$0 = -5(x-25)+10$

$\cancel{-10}^{\,2} = \cancel{-5}(x-25)$

$x - 25 = 2$

$\boxed{x = 27}$

Question 10 is printed on the next page.

10 **(a)** Solve $\tan 3x = -1$ for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ radians, giving your answers in terms of π. [4]

$-\frac{3\pi}{2} \le 3x \le \frac{3\pi}{2}$

$3x = \tan^{-1}(-1)$

$3x = -\frac{\pi}{4} + \pi k$

$3x = -\frac{\pi}{4}, \frac{3\pi}{4}, -\frac{5\pi}{4}$

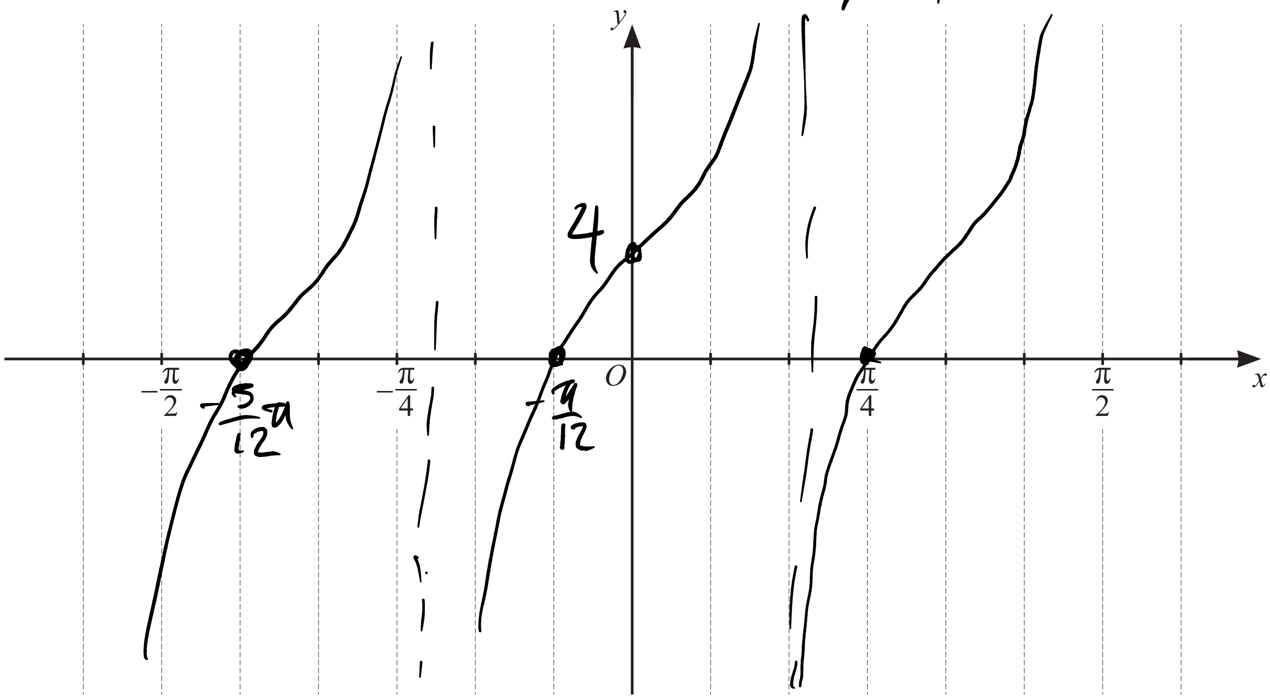
$$\boxed{x = -\frac{\pi}{12} \quad x = \frac{\pi}{4} \quad x = -\frac{5\pi}{12}}$$

(b) Use your answers to **part (a)** to sketch the graph of $y = 4\tan 3x + 4$ for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ radians on the axes below. Show the coordinates of the points where the curve meets the axes.

$4\tan(3x) + 4 = 0$

$\tan(3x) = -1$

$\frac{\pi}{8} + \frac{\pi}{3}k$



[3]